

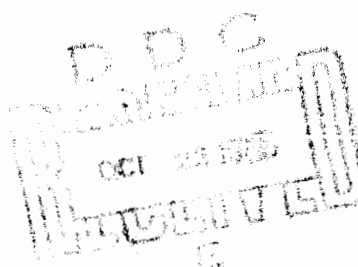
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NW Technical Memorandum

ATMOSPHERIC RETRACTION  
ON THE EFFECTIVE EARTH'S RADIUS:  
PROBLEMS AND PITFALLS

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January 1977



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FOREWORD

This technical memorandum is a copy of an article submitted for publication to the Institute for Electrical and Electronics Engineers (IEEE) Transactions on Antennas and Propagation. Some of the results presented herein plus additional results on geometry deformations due to atmospheric refraction has been submitted to the Union Radio-Scientifique Internationale (URSI) triennial Symposium on Electromagnetic Wave Theory, which is to be held in conjunction with the IEEE Internationale Symposium on Antennas and Propagation and the annual technical meeting of the U.S. National Committee of URSI. The Electromagnetic Wave Theory Symposium will be held on 20-24 June 1977 at Stanford University, Palo Alto, California. The work presented in this paper has been supported by a contract with the Naval Air Systems Command (AIR-370C).

This document has been prepared as an interim presentation of timely information. Although care has been taken in the preparation of the technical material presented, the results herein are to be considered as preliminary in nature.

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NWC TM 3028, published by Code 3173, 35 copies.

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ABSTRACT

↓  
The derivation of a transformation to compensate for refractivity in the earth's atmosphere is obtained through the use of ray tracing and differential geometry. The effective earth's radius, which is the classical method to compensate for the ray curvature as an electromagnetic wave propagates through the earth's atmosphere, is also derived and shown to be invalid since it does not serve its intended purpose -- namely, to conformally transform curved rays into straight lines. A brief review of atmospheric radio refractivity and ray tracing is presented and the differences between the refraction compensation transformation and the effective earth's radius are demonstrated. ↗



## NOMENCLATURE

C	curvature of the ray
c	speed of light in a vacuum ( $2.997925 \times 10^8 \text{ m}\cdot\text{s}^{-1}$ )
$c_o$	exponential refractivity constant
e	water vapor partial pressure
h	altitude above earth's surface
$h_1, h_2$	altitudes of the ray's end points
$K_1, K'_1, K_2, K_3, K_4, K_5$	refractivity constants
k	absorption index of refraction
L	effective spacing between neighboring rays
m	complex index of refraction
N	refractivity
$N_o$	refractivity at the earth's surface
n	phase index of refraction
$P_t$	total atmospheric pressure
$P_c$	carbon dioxide partial pressure
$P_d$	atmospheric pressure of dry air
r	distance from earth's center
$r_a$	equatorial radius of the earth
$r_b$	polar radius of the earth
$r_e$	radius of the earth
$r_k$	effective radius
$r_k^{(0)}$	effective earth's radius
$r_k^{(h)}$	effective earth's radius plus the altitude h

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$r_n$	radius of curvature of surface of constant refractivity
$r_s$	transformed radius
$r_1, r_2$	distances from earth's center to the ray's end points
$T$	atmospheric temperature
$x$	ray length
$\alpha$	refraction compensation transformation constant
$\nabla$	gradient operator
$\epsilon$	dielectric constant
$\Gamma$	ray factor
$\mu$	magnetic permeability
$\phi$	angle ray makes with surface of constant refractivity
$\phi_1, \phi_2$	angles that the ray makes with the surface of constant refractivity at the ray's end points
$\psi$	latitude on the earth's surface
$\rho$	radius of curvature of the ray
$\theta$	earth-center angle
$\theta_s$	transformed earth-center angle

## INTRODUCTION

From the Fermat's principle of least action, electromagnetic radiation (i.e., photons) always propagate along a geodesic in free space or through any homogeneous medium. However, if the radiation is propagating through an inhomogeneous medium (e.g., the earth's atmosphere), gradients in the speed of light in the medium cause the path of the photons, or ray, to deviate from a geodesic, or straight line. If the speed of light in the medium is discontinuous, the ray path will not be differentiable (e.g., at reflection points); however, for the earth's atmosphere it is reasonable to assume that the speed of light is a differentiable function of location. Then, the rays are twice differentiable curves through the atmosphere and hence have a curvature defined along the ray path.

This curvature of the ray paths due to inhomogeneities in the atmosphere causes considerable difficulties in attempting to solve various geometric problems. Although ray tracing is one solution, it is usually much too complex for most applications. Another technique has been the development of a transformation that maps the curved rays into straight lines and preserves with sufficient accuracy various geometric parameters such that the transformed geometry can be used to solve certain problems. The most common method used to compensate for the ray curvature in the earth's atmosphere is to use an effective earth's radius, which is usually larger than the true earth's radius, that supposedly conformally transforms the curved rays into straight lines. The treatment of ray curvature and the effective earth's radius in the standards of the literature has not been sufficiently rigorous due to either unnecessary or tacit assumptions [1] - [4]. This incompleteness leads to incorrect results due to improper use of the effective earth's radius. In addition, the effective earth's radius is not a conformal transformation, and it does not transform the curved rays into straight lines. Instead, the curved rays are transformed into rays with a greater radius of curvature.

In this paper, the basic theory of atmospheric radio refractivity is presented. Although the concepts of ray curvature and effective earth's radius are applicable for all frequencies, this paper concentrates on the propagation of microwave radiation. After a brief discussion of geometric optics, the effective earth's radius is derived and some of the associated errors are discussed. Finally, a transformation that compensates for the ray curvature is derived and compared with the effective earth's radius.



## REFRACTIVITY

The index of refraction of a medium is defined to be

$$m = \epsilon\mu = n - ik, \quad (1)$$

where  $\epsilon$  is the dielectric constant and  $\mu$  is the magnetic permeability of the medium. The real part of the index of refraction,  $n$ , called the phase index, is the ratio of the speed of light in a vacuum to that in the medium, while the imaginary part of the index of refraction is the absorption index,  $k$ . For the remainder of this report, the absorption index will be ignored. For electromagnetic radiation below 30 GHz in the earth's atmosphere, the phase index does not exceed unity by more than  $450 \times 10^{-6}$ , so a parameter called refractivity is defined by

$$N = (n-1) \times 10^6. \quad (2)$$

For frequencies below 30 GHz, the refractivity is given by Debye theory for polar and non-polar molecules as:

$$N = K_1 \frac{P_d}{T} + K_2 \frac{e}{T} + K_3 \frac{e}{T^2} + K_4 \frac{P_c}{T} \quad (3)$$

where  $P_d$  is the atmospheric pressure of dry air,  $P_c$  is the partial pressure of carbon dioxide,  $e$  is the partial pressure of water vapor, and  $T$  is the atmospheric temperature [1]. However, since in the troposphere (i.e., for altitudes less than 10-11 kilometers) the concentration of carbon dioxide by molecular weight is approximately 0.03 percent and  $K_4/K_1 = 5/3$ , it is customary to combine the pressure of dry air,  $P_d$ , with the partial pressure of carbon dioxide,  $P_c$ , resulting in

$$N = K'_1 \frac{P_t}{T} + K_2 \frac{e}{T} + K_3 \frac{e}{T^2}, \quad (4)$$

where  $P_t$  is the total atmospheric pressure. Experimental data results in the following values for the constants [1]:

$$\begin{aligned} K'_1 &= 77.607 \pm 0.13 & K/\text{mb} \\ K_2 &= 71.6 \pm 8.5 & K/\text{mb} \\ K_3 &= (3.747 \pm 0.031) \times 10^5 & K^2/\text{mb} \end{aligned}$$



Another common expression for the refractivity, which is sufficiently accurate for many applications, is [1]

$$N = K_1' \frac{P}{T} + K_5 \frac{e}{T^2} \quad (5)$$

where

$$K_5 = 3.73 \times 10^5 \quad K^2/\text{mb}$$

The constants in (5) are considered to be accurate to within 0.5 percent in  $N$  for frequencies up to 30 GHz and normally encountered ranges of pressure, temperature and humidity [1]. The first term of (5) is usually referred to as the "dry" term and the second term is referred to as the "wet" term. As can be seen from (4) and (5), the propagation of microwave energy through the earth's atmosphere is affected primarily by gradients in the wet term.

#### ATMOSPHERIC REFRACTIVITY MODELS

Although the expressions for refractivity as a function of pressure, temperature, and water vapor partial pressure are essential if one desires to evaluate electromagnetic wave propagation under particular atmospheric conditions, it is frequently convenient to use a model that represents the mean refractivity profile for a given location or region. Several models in common use today are discussed below. It should be mentioned, though, that the deviation from the mean may be quite significant and the profile that is referred to as "normal" may occur only a small percentage of the time.

##### LINEAR ATMOSPHERE

The linear model assumes that the mean refractivity varies linearly with the altitude; i.e.,

$$N = N_0 + h \frac{dN}{dh}, \quad (6)$$

where  $N_0$  is the mean refractivity at zero altitude, and  $h$  is the altitude. This model can only be considered valid for low altitudes (i.e., less than 1 kilometer). Standard conditions are considered to be  $N_0 = 301.0$  and  $\frac{dN}{dh} = 39.25 \text{ km}^{-1}$ .

## EXPONENTIAL ATMOSPHERE

In this model, the mean refractivity varies exponentially with the altitude; i.e.,

$$N = N_0 e^{-c_0 h}, \quad (7)$$

where  $c_0$  is a constant. This model gives fairly close agreement with the mean refractivity for altitudes less than 3 kilometers and is still moderately close for much higher altitudes. The exponential atmosphere has been adopted by the National Bureau of Standards with values of  $N_0$  and  $c_0$  given in Table 1 [1]. With these values the exponential model is referred to as the Central Radio Propagation Laboratory (CRPL) Exponential Reference Atmosphere (1958).

## OTHER MEAN ATMOSPHERES

Other model atmospheres have been developed to represent the mean refractivity as a function of altitude. One such model uses various combinations of linear and exponential atmospheres for different altitude ranges. Another model uses a biexponential atmosphere, in which the wet and dry terms of (5) are assumed to vary exponentially but with different scale height constants. These model atmospheres can be found in the Reference [1].

## NON-STANDARD ATMOSPHERES

The use of mean and standard atmospheres has created much erroneous data and misconceptions concerning the propagation of microwave radiation in the earth's atmosphere since many operational personnel have treated the standard atmosphere as the most likely one. Non-standard atmospheres which involve various anomalous effects are quite common in various parts of the world and have a significant effect upon the propagation of electromagnetic waves [5]. A study of these non-standard atmospheres is beyond the scope of this paper; however, certain microwave propagation conditions can be classified:

Sub refraction:	$0.0 < \Gamma$
Normal refraction:	$-0.5 < \Gamma < 0.0$
Superrefraction:	$-1.0 < \Gamma < -0.5$
Trapping (ducting):	$\Gamma < -1.0,$

where  $\Gamma$  is the ray factor defined by

$$\Gamma = \frac{r}{n} \frac{dn}{dr_n}, \quad (8)$$

TABLE 1. Table of Constants for the CRPL Exponential  
Radio Refractivity Atmosphere (1958) [1].

$N_o$	$c_o \text{ (km}^{-1}\text{)}$
0.0	0.000000
200.0	0.118400
250.0	0.125625
252.9	0.126255
301.0	0.139632
313.0	0.143859
344.5	0.156805
350.0	0.159336
377.2	0.173233
400.0	0.186720
404.9	0.189829
450.0	0.223256



where  $n$  is the phase index of refraction, which is assumed to be a differentiable function of  $r_n$ , and  $r_n$  is the radius of curvature of the surface of constant refractivity. For a spherically stratified atmosphere  $r_n = r$ , the distance from the earth's center. It should be noted that this classification distinguishes between superrefraction and trapping. This distinction is not always made in the literature; however, due to the significant difference in propagation characteristics, it is felt that the distinction should be made.

#### GEOMETRIC OPTICS (RAY TRACING)

Since the concepts of geometric optics are used in developing the theory of the effective earth's radius, it is useful to evaluate when geometric optics, or ray tracing, is valid and to give some expressions for ray tracing in the earth's atmosphere. For a more detailed treatment of the subject, the reader is referred to Stavroudis [6], an excellent mathematical treatment of ray theory. Ray tracing is a valid approximation to the propagation of electromagnetic waves through a medium if the phase index of refraction does not change significantly within a distance comparable to the wavelength of the signal (i.e.,  $|\nabla n| \ll 2\pi n^2 \lambda$ ) and if the relative change in the effective spacing,  $L$ , between neighboring rays does not change significantly with respect to the wavelength of the signal (i.e.,  $|\nabla L| \ll 2\pi n L / \lambda$ ). Both of these assumptions are valid for non-trapping atmospheric conditions. If trapping occurs, special care must be taken to avoid caustics and various diffraction phenomena.

The assumption is made for the remainder of this paper that the phase index of refraction is a differentiable function that depends upon altitude only. The assumption that  $n$  depends upon  $r$  only (i.e., the atmosphere is spherically stratified, or horizontally homogeneous) is a fairly conventional one; although this assumption is not necessary for the development of ray tracing or the effective earth's radius, it simplifies the mathematics somewhat. The assumption that  $n$  is a differentiable function is not too stringent a condition; it may simply require some smoothing of experimental data.

Using Snell's Law,

$$nr_n \cos \phi = \text{constant}, \quad (9)$$

and some simple differential geometry, the above assumptions quickly give rise to the following relations for a ray propagating through a horizontally homogeneous atmosphere:

$$\frac{dr}{d\phi} = \frac{r \tan \phi}{1+\Gamma}, \quad (10)$$

$$\frac{d\theta}{d\phi} = \frac{1}{1+\Gamma}, \quad (11)$$

$$\frac{dr}{d\theta} = r \tan \phi, \quad (12)$$

$$\frac{dx}{dr} = \frac{n}{\sin \phi}, \quad (13)$$

$$C = \frac{1}{\rho} = \frac{-\Gamma \cos \phi}{r}, \quad (14)$$

where  $r$  is the distance from the earth's center,  $\phi$  is the angle between the ray and the surface of constant refractivity,  $\theta$  is the earth-center angle,  $x$  is the distance along the ray between the ray's end points,  $C$  is the ray curvature,  $\rho$  is the radius of curvature of the ray and  $\Gamma$  is the ray factor.

It should be noted that for this paper, the actual value of the earth's radius is not of great importance. This discussion is concerned only with rays propagating between two points in the earth's atmosphere and the interaction of the electromagnetic wave with the earth's surface is not considered. Therefore, all of the analysis herein is valid for an atmosphere whose surfaces of constant refractivity are concentric spheres and whose radii may have nothing whatsoever to do with the radius of the earth. However, for reference and for numerical data presented near the end of this paper, the appendix contains a brief discussion of the radius of the earth.

#### EFFECTIVE EARTH'S RADIUS

The classical approach to compensate for the curvature of ray paths due to atmospheric refraction is to assume that the earth's radius is somewhat larger than it actually is. This method was advanced by Schelleng, Burrows and Ferrell [7] and has been used by many authors since. The concept is to absorb the curvature of the ray into the curvature of the effective earth so that the relative curvature remains the same. If  $r$  is the distance from a point on the ray to the center of the earth and  $\rho$

is the radius of curvature of the ray at that point, the effective radius  $r_k$  is defined to be

$$r_k = [r^{-1} - \rho^{-1}]^{-1} . \quad (15)$$

Since any twice differentiable curve has a radius of curvature associated with each point on the curve, this technique applies to any realistic ray path (i.e., if  $n$  is differentiable). The negative sign results from the convention that a downward curving ray has positive curvature and an upward curving ray has negative curvature. This results in an effective radius of

$$r_k = \frac{r}{1 + \Gamma \cos \phi} . \quad (16)$$

Equation (16) is applicable for any atmospheric condition in which ray tracing is valid. However, both  $\Gamma$  and  $\phi$  are functions of the position of the point along the ray path; therefore,  $\Gamma \cos \phi$  varies along the ray path. For this reason, the effective radius is useful if and only if the variation in  $\Gamma \cos \phi$  along the ray path is small and can be ignored. However, it should be pointed out that at low altitudes for exponential and linear atmospheres,  $r_k$  decreases with increasing altitude.

Many authors have treated the effective earth's radius using assumptions that are not always explicitly stated. The effective earth's radius,  $r_k^{(0)}$ , is defined as the effective radius evaluated at the earth's surface with  $\phi = 0$ , so

$$r_k^{(0)} = \frac{r_e}{1 + \Gamma} , \quad (17)$$

where  $r_e$  is the radius of the earth. It is then claimed that the effective radius that should be used is

$$r_k^{(h)} = r_k^{(0)} + h, \quad (18)$$

since  $r = r_e + h$ . It should be noted that there is significant difference between  $r_k$  and  $r_k^{(h)}$ . For normal propagation,  $\Gamma = -1/4$ , so  $r_k^{(0)} = \frac{4}{3} r_e$ , the so-called 4/3-earth model. If normal propagation is



assumed, for small look angles ( $\phi = 0$ ) differences of 33% exist between the altitude difference of the ray's end points calculated using the effective earth's radius and that using the effective radius; for a very high look angle ( $\phi = \pi/2$ ), the difference is approximately zero. Also, due to the high chances of subrefraction, superrefraction, and trapping propagation conditions in certain areas of the world, other values of  $\Gamma$  may be necessary. Depending upon the atmospheric conditions, the actual differences might be less or greater than those above.

It is often assumed that the effective earth's model is applicable only when the linear atmosphere is valid. However the only requirement for applicability is that  $\Gamma$  be relatively uniform over the ray path between the transmitter and the receiver, and the model of atmospheric refractivity is immaterial.

Using the effective earth's radius, it is assumed that all transformed rays can be represented as straight lines over the surface of the effective earth and that the transformation is conformal (i.e., angles are preserved). However, if one examines the transformed rays, it becomes readily apparent that they are not straight lines, and the transformation is not conformal. For example, since the effective earth's radius transformation does not alter the earth's center angle between the transmitter and the receiver, it is impossible to draw a triangle that also preserves the angles,  $\phi_1$  and  $\phi_2$ , between the ray and the surfaces of constant refractivity at the transmitter and receiver, respectively; therefore, if the transformed rays were straight, the transformation would not be conformal. Also, a detailed study of the transformed rays show that although their curvature has decreased, it is not zero (i.e., it is not a straight line). In the following section, a transformation that maps the curved rays into straight lines is derived and the differences between that transformation and both the effective earth's radius and the effective radius transformations are demonstrated.

#### REFRACTION COMPENSATION TRANSFORMATION

The curvature of a twice differentiable curve in spherical coordinates is defined to be

$$C = \frac{r^2 + 2\dot{r}^2 - r\ddot{r}}{(r^2 + \dot{r}^2)^{3/2}}, \quad (19)$$

where  $\dot{r}$  implies differentiation with respect to  $\theta$ . Requiring that the angle  $\phi$  be preserved all along the ray, and using differential geometry, the condition that the transformed ray is a straight line (i.e., the curvature is zero) is satisfied by

$$r^2 + 2\dot{r}^2 = r \ddot{r}, \quad (20)$$

which is satisfied if and only if

$$\frac{dr}{dr} = \frac{r}{r} (1+\Gamma) \quad (21a)$$

$$\left(\frac{d\theta}{d\phi}\right)^2 = 1. \quad (21b)$$

Solving these equations and using simple geometry, the refraction compensation transformation is defined as follows:

$$r_s = \alpha nr \quad (22a)$$

$$\theta_s = \phi - \phi_1, \quad (22b)$$

where  $(r_s, \theta_s)$  is the transformed polar coordinate representation of the point  $(r, \theta)$ ,  $\phi$  is the look angle at the point  $(r, \theta)$ ,  $\phi_1$  is the look angle at the point  $(r_1, 0)$ , and  $\alpha$  is some arbitrary constant. The value of the constant  $\alpha$  depends upon the application of the transformation. For example, if it is desired that the transformed ray length is equal to the true ray length, then

$$\alpha = \left[1 + \frac{\int r dx}{\int dx}\right]^{-1} \quad (23)$$

where the line integrals are evaluated along the ray. Therefore, the parameter  $\alpha$  is dependent upon the ray direction, or look angle,  $\phi$ . To analyze the validity of certain approximations to  $\alpha$  for all rays, two atmospheric regions for exponential atmospheres were studied - namely, the region between the surface and 1 kilometer, which can be considered as fairly linear in nature, and the region between the surface and 10 kilometers, where the exponential nature of refractivity becomes more apparent.

Recognizing that

$$\frac{d}{dr} (nr) = n (1+\Gamma), \quad (24)$$

it is possible to approximate  $\alpha$  for rays propagating between altitudes  $r_1$  and  $r_2$  by

$$\alpha \approx \frac{r_2 - r_1}{n_2 r_2 - n_1 r_1} \quad (25)$$

Another approximation for  $\alpha$  is given by

$$\alpha \approx [1 + \frac{\Gamma_1 + \Gamma_2}{2}]^{-1} \quad (26)$$

where  $\Gamma_1$  and  $\Gamma_2$  are the ray factors at  $r_1$  and  $r_2$ , respectively.

Equation (26) has errors comparable to or less than those of (25) for angles less than  $1^\circ$ , but (25) is a better approximation for angles above  $1^\circ$ , especially for the surface to 10 kilometer region. A comparison between the two approximations is given in Table 2. It should be noted that the errors given in Table 2 are for unstratified atmospheres, such that  $-0.5 \leq \Gamma \leq 0.0$  (i.e., normal propagation), where the refractivity can be approximated either by a linear or by an exponential fit. The larger errors generally occur for the smaller values of  $\Gamma$ ; this implies that extreme care should be taken when using these approximations in superrefracting atmospheres unless the angle is significantly large (i.e., larger than  $10^\circ$ ).

Several remarks are in order concerning the refraction compensation transformation:

a. The factor of  $n$  in the radial component of the transformation is crucial. Although  $n$  is very close to unity, the gradient of  $n$  is comparable with  $r^{-1}$  so that this factor is important and should not be deleted.

b. The earth-center angle,  $\theta$ , does not appear directly in the transformed coordinates. Therefore, extreme care should be taken in the use of the transformed earth-center angle,  $\theta_s$ . This difference between  $\theta$  and  $\theta_s$  is not considered in the effective radius or effective earth's radius transformations.

c. The radius of the earth does not enter directly into the calculations of the transformed coordinates. The parameter  $r$  equals  $r_e + h$  and for a spherically stratified atmosphere is the radius of curvature of the surface of constant refractivity. This is the most serious defect in the effective earth's radius model. The same relative errors in altitude exist between the effective earth's radius and the



TABLE 2. Comparison of Approximations to Constant  $\alpha$ .

$\phi$	0 - 1 km		0 - 10 km	
	Equation 25	Equation 26	Equation 25	Equation 26
0.0°	0.88-2.84%	0.71-3.48%	3.99-9.52%	0.40-2.02%
0.5°	0.29-0.79%	0.12-1.43%	2.58-5.85%	0.29-3.12%
1.0°	0.14-0.32%	0.04-0.97%	1.82-3.98%	0.41-4.9%
10.0° (*)	0.01-0.04%	0.06-0.87%	0.10-0.18%	2.17-8.64%

\*Note:  $\alpha$  is fairly constant for angles greater than 10°.

refraction compensation transformations as between the effective earth's radius and the effective radius transformations (i.e., approximately  $\frac{r}{1+r}$  ).

d. Although the evaluation of the parameter  $\alpha$  does depend slightly upon angle  $\phi$ , the dependence is nowhere near as great as indicated by the effective radius transformation, which is inaccurate except for low angles.

#### CONCLUSIONS

The effective earth's radius model, although in use for many years, is essentially in error. The techniques used to derive it do not produce a transformation that maps rays into straight lines. As a result, it is possible to obtain significant errors under certain applications. The refraction compensation transformation does produce transformed rays which have zero curvature. Also, the effects of the transformation on the earth-center angle, which are ignored in the effective earth's radius model, can be significant. In general, the author feels that the use of an effective earth's radius contains enough errors to justify its removal from use in the literature. The refraction compensation transformation presented in this paper is recommended to replace the effective earth's radius transformation wherever possible.

#### ACKNOWLEDGEMENTS

The author would like to thank the Naval Air Systems Command (AIR-370C), which supported the work presented in the paper, and Dr. Alexis Shlanta, who provided much valuable information on atmospheric physics.

## Appendix

## RADIUS OF THE EARTH

Generally, the earth can be considered as an oblate spheroid (i.e., a sphere flattened at the poles). Due to a non-uniform mass-distribution, the direction of the gravity is tilted away from the center of the earth; the shape of the earth whose surface is normal to the direction of gravity is called a *geoid*. Over the land the radius of the spheroid is generally less than the radius of the geoid, while over the sea the opposite is generally true. Due to slight irregularities in the shape of the earth, several different models exist for the size of the earth. The four values usually given are:

- (a) Equatorial radius,  $r_a$ ;
- (b) Polar radius,  $r_b$ ;
- (c) Mean radius,  $(2r_a + r_b)/3$ .
- (d) Ellipticity or flattening,  $e = (r_a - r_b)/r_a$ .

The more commonly occurring values for the above values are given in Table AI. The various models are:

(i) *Clarke Spheroid of 1866*. Computed by the English geodesist, A. R. Clarke, this model is used for charts of North America. Since Clarke did not clearly define his units, the U.S. Coast and Geodetic Survey adjusted Clarke's values in 1880 by adding approximately 85 feet to obtain the standard values given in Table AI, which are used for all charts and maps of North America.

(ii) *Clarke Spheroid of 1880*. These values are new estimates made by Clarke in 1880; however, they have not been adopted by the United States.

(iii) *International Spheroid*. In 1909-10, Hayford [8] conducted measurements such that the error in the equatorial radius is  $\pm 18$  meters and the error in  $e^{-1}$  is  $\pm 0.5$ . These values are considered to be a slight improvement over either of Clarke's models. However, Helmet [9] increased these errors to  $\pm 53$  meters and  $\pm 1.2$ , respectively.

(iv) *Sir Harold Jeffreys (1952 and 1970)*. Jeffreys [10] is a standard reference for parameters concerning the earth; however, the values given therein are not referenced to any experiment.



TABLE A1. Earth Radii.

	Equatorial Radius (m)	Polar Radius (m)	Mean Radius (m)	Ellipticity
Clarke Spheroid of 1866	6378206.4	6356583.8	6370998.9	$(294.98)^{-1}$
Clarke Spheroid of 1880	6378249.145	6356514.870	6371004.387	$(293.465)^{-1}$
International Spheroid	6378388.000	6356911.946	6371299.315	$(297)^{-1}$
Jeffreys (3rd Edition)	6378099.	6356641.	6370943.	$(297)^{-1}$
Jeffreys (5th Edition)	6378269 (1+u') *	-	-	$0.00336700+e'$ *

\* u' and e' are small corrections; good estimates are  $u'=e'=0$

For most applications, the mean radius of the International Spheroid is usually adequate. If a more accurate value is necessary, for a latitude  $\psi$ , assuming an elliptical earth, use

$$r_e = \sqrt{r_a^2 \cos^2 \psi + r_b^2 \sin^2 \psi} \quad (A-1)$$

REFERENCES

1. B. R. Bean and E. J. Dutton, *Radio Meteorology*. New York: Dover, 1968. 435 pp.
2. B. R. Bean, E. J. Dutton, and B. D. Warner, "Weather Effects on Radar", *Radar Handbook*, M. I. Skolnik (Ed.). New York: McGraw-Hill 1970.
3. C. R. Burrows and S. S. Attwood, *Radio Wave Propagation*. New York: Academic Press, 1949. 548 pp.
4. D. E. Kerr, (Ed.) *Propagation of Short Radio Waves*. New York: Dover, 1965. 729 pp.
5. B. R. Bean et al. *A World Atlas of Atmospheric Radio Refractivity*. Washington DC: Environmental Science Services Administration, 1966. Monograph 1. 130 pp.
6. O. N. Stavroudis, *The Optics of Rays, Wavefronts, and Caustics*. New York: Academic Press, 1972. 313 pp.
7. J. C. Schelleng, C. R. Burrows, and E. B. Ferrell, "Ultra-short-wave Propagation", *Proc. IRE*, Vol. 21, pp. 427-463, 1933.
8. J. F. Hayford, *The Figure of the Earth and Isotasy and Supplementary Investigation in 1909*. Washington DC: U.S. Coast and Geodetic Survey, 1909-1910.
9. F. R. Helmut, *S. B. preuss Akad. Wiss.*, pp. 10-19, 1911.
10. Sir Harold Jeffreys, *The Earth: Its Origins, History, and Physical Constitution*, 5th ed. London: Cambridge Univ. Press, 1970. 525 pp.